QUESTION 2007

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following:

i) The differential coefficient of x^6 with respect to x^3 is

✓a) $2x^3$ b) $2x$ c) $2x^2$ d) 2

ii) If $y = \log x^2$, the value of $\frac{d^2y}{dx^2}$

a) $\frac{2}{x^3}$ ✓b) $-\frac{2}{x^2}$ c) $\frac{2}{x}$ d) $2x$

iii) If $y = 2$ at and $x = at^2$, then $\frac{dy}{dx}$ at $t = 1$ is

✓a) 1 b) 2a c) –1 d) $2a^2$

iv) The value of $\int \text{cosec}^2 2x\, dx$ is

✓a) $-\frac{1}{2}\cot 2x$ b) cot 2x c) $\frac{\text{cosec}\, 2x}{2}$ d) – cot 2x

v) If $P = \{2, 4, 6, 7, 8, 9\}$, $Q = \{1, 2, 6, 9\}$, then $P - Q$ is

✓a) {4, 7, 8} b) {4, 6, 8, 9} c) {1} d) {2, 4, 6, 7, 8, 9}

vi) The value of t for which the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 5 & t & 3 \\ 0 & 3 & 1 \end{bmatrix}$ is singular is

a) $-\frac{3}{2}$ b) 2 ✓c) $\frac{3}{2}$ d) -2

vii) The value of $\lim_{x \to 0} \frac{\sin x}{x}$, (where x is radian) is

✓a) 1 b) 0 c) ∞ d) -1

viii) The value of $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ is

a) 1 ✓b) 4 c) 0 d) 2

ix) If $A = \{1, 2, 3\}$, $B = \{a, b\}$, $A \times B$ is given by

✓a) {(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)} b) {(a, 1), (a, 2), (a, 3), (b,1), (b, 2), (b, 3)}

c) {(1, a), (1, b), (2, a), (2, b), (3, a), (3, b), ( )} d) {1, 2, 3, a, b}

x) Solution of the equation $x^3 + 2x + 3 = 0$ will give us

✓a) no real positive roots but one real negative root

b) 2 real positive roots and 1 real negative root

c) 1 real positive root and 2 imaginary roots

d) 2 real negative roots and only one imaginary root

xi) Matrix A and Matrix B of order m × n are said to be equal, if

a) m = n b) $A^T = B^T$ ✓c) (A – B) equals the null matrix d) AB = BA

xii) Rank of matrix $A = \begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix}$ is given by

✓a) 2 b) 1 c) -1 d) 0

Group – B

(Short Answer Type Questions)

2. Evaluate $\int \frac{1}{x^2} e^{\frac{1}{x}} dx$

See Topic: INDEFINITE INTEGRATIONS, Short Answer Type Question No. 7.

3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 & 2 \\ 5 & -3 & -1 \end{bmatrix}$, then find A + B and AB.

See Topic: LIMIT, CONTINUITY & DIFFERENTIABILITY, Short Answer Type Question No. 7.

4. A function f(x) is defined as follows

$f(x) = -x$ when $x \leq 0$

$= x$ when $0 < x < 1$

$= 2 - x$ when $x \geq 1$

Show that the f(x) is continuous at x = 0 and x =1.

See Topic: LIMIT, CONTINUITY & DIFFERENTIABILITY, Long Answer Type Question No. 6.

5. If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

See Topic: FUNCTION OF SEVERAL VARIABLES, Long Answer Type Question No. 4.

6. If a, b are the roots of the equation $x^2 - px + q = 0$, then find the equation whose roots are 1/a and 1/b.

See Topic: POLYNOMIAL, Short Answer Type Question No. 11.

7. Solve the following system of equations using Cramer's rule:

$x + 2y + 3z = 6$

$2x + 4y + z = 7$

$3x + 2y + 9z = 14$

See Topic: MATRICES, Long Answer Type Question No. 7.

Group – C
(Long Answer Type Questions)

8. a) Solve the following equation by matrix inversion method:

$$x + y + z = 8$$
$$x - y + 2z = 6$$
$$3x + 5y + 7z = 14$$

b) Give the definition of commutative group and show that $\{1, \omega, \omega^2\}$ where $\omega^3 = +1$ forms a commutative group in respect of multiplication.

a) See Topic: MATRICES, Long Answer Type Question No. 8.

b) See Topic: BINARY COMPOSITION, Short Answer Type Question No. 3.

9. a) In a class of 50 students, 15 read Physics, 20 read Chemistry and 20 read Mathematics. 3 read Physics and Chemistry, 6 read Chemistry and Mathematics and 5 read Physics and Mathematics. 7 read none of the subjects. How many studefits read all the subjects?

b) Integrate $\int xe^x/(x+1)^2\,dx$.

a) See Topic: SET THEORY, Long Answer Type Question No. 3.

b) See Topic: INDEFINITE INTEGRATIONS, Short Answer Type Question No. 8.

10. a) Find dy/dx when $x = y \log(xy)$

b) Find for what values of x, the following expression is maximum and minimum respectively: $2x^3 - 21x^2 + 36x - 20$.

c) A function f(x) is defined in the following way:

$f(x) = -x$ when $x \le 0$
$= x$ when $0 < x < 1$
$= 2 - x$ when $x \ge 1$

Show that it is continuous at $x = 0$.

a) See Topic: SUCCESSIVE DIFFERENTIATION, Short Answer Type Question No. 7.

b) See Topic: MISCELLANEOUS, Long Answer Type Question No. 2.

c) See Topic: LIMIT, CONTINUITY & DIFFERENTIABILITY, Long Answer Type Question No. 7.

11. a) Evaluate $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx$

b) Evaluate $\int_0^{\pi/2} \log(\tan x)\,dx$

c) If A = {a, b, c, d, e}, B = {c, a, e, g} and C = {b, e, f, g}, then show that $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

POPULAR PUBLICATIONS

a) See Topic: DEFINITE INTEGRALS, Short Answer Type Question No. 3.
b) See Topic: DEFINITE INTEGRALS, Short Answer Type Question No. 5.
c) See Topic: SET THEORY, Long Answer Type Question No. 4.

12. a) Discuss the nature of the conic represented by $3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0$ by reducing to its canonical form.

b) Find X from the matrix equation AX = B, where $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 1 \\ 7 \end{bmatrix}$

c) If $f(x, y) = \log(x^2y + xy^2)$ find f_{xx}, f_{xy}, f_{yy} and f_{yx}.

a) See Topic: GENERAL EQUATION OF SECOND DEGREE, Long Answer Type Question No. 2.
b) See Topic: MATRICES, Long Answer Type Question No. 9.
c) See Topic: FUNCTION OF SEVERAL VARIABLES, Long Answer Type Question No. 5.

13. a) Apply Descarte's rule of sign to show that the equation $x^4 + 2x^2 - 7x - 5 = 0$ has two real roots and two non – real roots.

b) Find the maximum and minimum values of f(x) where $f(x) = x + \frac{1}{x}$

c) Write Taylor's formula for the function $f(x) = e^x$ about the point x = 0 with Lagrange's form of remainder after 3 terms.

a) See Topic: POLYNOMIAL, Long Answer Type Question No. 7.
b) See Topic: MISCELLANEOUS, Short Answer Type Question No. 2.
c) See Topic: EXPANSION OF FUNCTION, Long Answer Type Question No. 2.